3.7.4.2 Wind turbine flow states

Measured wind turbine performance closely approximates the results of BEM theory at low values of the axial induction factor. Momentum theory is no longer valid at axial induction factors greater than 0.5, because the wind velocity in the far wake would be negative. In practice, as the axial induction factor increases above 0.5, the flow patterns through the wind turbine become much more complex than those predicted by momentum theory. A number of operating states for a rotor have been identified (see Eggleston and Stoddard, 1987). The operating states relevant to wind turbines are designated the windmill state and the turbulent wake state. The windmill state is the normal wind turbine operating state. The turbulent wake state occurs under operation in high winds. Figure 3.26 illustrates fits to measured thrust coefficients for these operating states. The windmill state is characterized by the flow conditions described by momentum theory for axial induction factors less than about 0.5. Above \( \alpha = 0.5 \), in the turbulent wake state, measured data indicate that thrust coefficients increase up to about 2.0 at an axial induction factor of 1.0. This state is characterized by a large expansion of the slipstream, turbulence and recirculation behind the rotor. While momentum theory no longer describes the turbine behavior, empirical relationships between \( C_T \) and the axial induction factor are often used to predict wind turbine behavior.

3.7.4.3 Rotor modelling for the turbulent wake state

The rotor analysis discussed so far uses the equivalence of the thrust forces determined from momentum theory and from blade element theory to determine the angle of attack at the blade. In the turbulent wake state the thrust determined by momentum theory is no longer valid. In these cases, the previous analysis can lead to a lack of convergence to a solution or a situation in which the curve defined by Equation 3.7.6a or 3.7.6 would lie below the airfoil lift curve.

In the turbulent wake state, a solution can be found by using the empirical relationship between the axial induction factor and the thrust coefficient in conjunction with blade element theory. The empirical relationship developed by Glauert, and shown in Figure 3.26, (see Eggleston and Stoddard, 1987), including tip losses, is:
Figure 3.26  Fits to measured wind turbine thrust coefficients

\[
\alpha = \left( \frac{\|F\|}{C_T} \right) \left[ 0.143 + \sqrt{0.0203 - 0.6427(0.889 - C_T)} \right] \\
(3.7.21)
\]

This equation is valid for \( \alpha > 0.4 \) or, equivalently for \( C_T > 0.96 \).

The Glauert empirical relationship was determined for the overall thrust coefficient for a rotor. It is customary to assume that it applies equally to equivalent local thrust coefficients for each blade section. The local thrust coefficient, \( C_{T_r} \), can be defined for each annular rotor section as (Wilson et al., 1976):

\[
C_{T_r} = \frac{dF_N}{\frac{1}{2} \rho U^2 2\pi r dr} \\
(3.7.22)
\]

From the equation for the normal force from blade element theory, Equation 3.7.1, the local thrust coefficient is:

\[
C_{T_r} = \sigma^2 (1 - \alpha)^2 (C_a \cos \varphi + C_d \sin \varphi) \sin^2 \varphi \\
(3.7.23)
\]

The solution procedure can then be modified to include heavily loaded turbines. The easiest procedure to use is the iterative procedure (Method 2) that starts with the selection of possible values for \( \alpha \) and \( \alpha' \). Once the angle of attack and \( C_I \) and \( C_d \) have been determined, the local thrust coefficient can be calculated according to Equation 3.7.23. If \( C_{T_r} < 0.96 \) then the previously derived equations can be used. If \( C_{T_r} > 0.96 \) then the next estimate for the axial induction factor should be determined using the local thrust coefficient and Equation 3.7.21. The angular induction factor, \( \alpha' \), can be determined from 3.7.24.

\[
\alpha' = \left( \frac{\|F\|}{C_T} \right) \left[ 0.143 + \sqrt{0.0203 - 0.6427(0.889 - C_T)} \right] \\
(3.7.24)
\]
3.7.4.4 Off-axis flows and blade coning

The analysis in this chapter assumes that the prevailing wind is uniform and aligned with the rotor axis and that the blades rotate in a plane perpendicular to the rotor axis. These assumptions are rarely the case because of wind shear, yaw error, vertical wind components, turbulence and blade coning. Wind shear will result in wind speeds across the disk that vary with height. Wind turbines often operate with a steady state or transient yaw error (misalignment of the rotor axis and the wind direction about the vertical yaw axis of the turbine). Yaw error results in a flow component perpendicular to the rotor disk. The winds at the rotor may also have a vertical component, especially at sites in complex terrain. Turbulence results in a variety of wind conditions over the rotor. The angular position of the blade in the rotor plane is called the azimuth angle and is measured from some suitable reference. Each of the effects mentioned above results in conditions at the blade varying with blade azimuth angle. Finally, blades are also often attached to the hub at a slight angle to the plane perpendicular to the rotor axis. This blade coning may be done to reduce bending moments in the blades or to keep the blades from striking the tower.

In a rotor analysis, each of these situations is usually handled with appropriate geometrical transformations. Blade coning is handled by resolving the aerodynamic forces into components that are perpendicular and parallel to the rotor plane. Off-axis flow is also resolved into the flow components that are perpendicular and parallel to the rotor plane. Rotor performance is then determined for a variety of rotor azimuth angles. The axial and in-plane components of the flow that depend on the blade position result in angles of attack and aerodynamic forces that fluctuate cyclically as the blades rotate. BEM equations that include terms for blade coning are provided by Wilson et al. (1976). Linearized methods for dealing with small off-axis flows and blade coning are discussed in Chapter 4.

3.8 Blade Shape for Optimum Rotor with Wake Rotation

The blade shape for an ideal rotor that includes the effects of wake rotation can be determined using the analysis developed for a general rotor. This optimisation includes wake rotation, but ignores drag ($C_d = 0$) and tip losses ($F = 1$). One can perform the optimisation by taking the partial derivative of that part of the integral for $C_p$ (Equation 3.7.12) which is a function of the angle of the relative wind, $\varphi$, and setting it equal to zero, i.e.:

$$\frac{\partial}{\partial \varphi} \left[ \sin^2 (\cos \varphi - \lambda_r \sin \varphi) (\sin \varphi + \lambda_r \cos \varphi) \right] = 0$$

This yields:

$$\lambda_r = \sin \varphi (2 \cos \varphi - 1) / \left[ (1 - \cos \varphi)(2 \cos \varphi + 1) \right]$$
Some more algebra reveals that:

\[ \varphi = \left( \frac{2}{3} \right) \tan^{-1} \left( \frac{1}{\lambda_r} \right) \]  
(3.8.3)

\[ c = \frac{8\pi r}{BC_l} \left( 1 - \cos \varphi \right) \]  
(3.8.4)

Induction factors can be calculated from:

\[ a' = \frac{1-3a}{4a-1} \]  
(3.3.17)

\[ a = \sqrt[4]{1 + 4 \sin^2 \varphi} \left( \frac{\sigma C_l \cos \varphi}{C} \right) \]  
(3.7.9)

These results can be compared with the result for an ideal blade without wake rotation, for which:

\[ \varphi = \tan^{-1} \left( \frac{2}{3\lambda_r} \right) \]  
(3.6.7)

\[ c = \frac{8\pi r}{BC_l} \left( \frac{\sin \varphi}{3\lambda_r} \right) \]  
(3.6.8)

Note, that the optimum values for \( \varphi \) and \( c \), including wake rotation, are often similar to, but could be significantly different from, those obtained without assuming wake rotation. Also, as before, select \( \alpha \) where \( C_d/C_l \) is minimum.

Solidity is the ratio of the area of the blades to the swept area, thus:

\[ \sigma = \frac{1}{nR^2} \int_{R_h}^{R} c \, dr \]  
(3.8.5)

The optimum blade rotor solidity can be found from methods discussed above. When the blade is modelled as a set of \( N \) blade sections of equal span, the solidity can be calculated from:

\[ \sigma \equiv \frac{B}{N\pi} \left( \sum_{i=1}^{N} \frac{c_i/R}{R} \right) \]  
(3.8.6)
The blade shape for three sample optimum rotors, assuming wake rotation, are given in Table 3.3. Here \( C_{t1} \) is assumed to be 1.00 at the design angle of attack. In these rotors, the blade twist is directly related to the angle of the relative wind because the angle of attack is assumed to be constant (see Equations 3.5.4 and 3.5.5). Thus, changes in blade twist would mirror the changes in the angle of the relative wind shown in Table 3.3. It can be seen that the slow 12 bladed machine would have blades that had a roughly constant chord over the outer half of the blade and smaller chords closer to the hub. The blades would also have a significant twist. The two faster machines would have blades with an increasing chord as one went from the tip to the hub. The blades would also have significant twist, but much less than the 12 bladed machine. The fastest machine would have the least twist, which is a function of local speed ratio only. It would also have the smallest chord because of the low angle of the relative wind and only two blades (see Equations 3.8.3 and 3.8.4).

### Table 3.3  Three optimum rotors

<table>
<thead>
<tr>
<th>( \frac{r}{R} )</th>
<th>( \lambda = 1 )</th>
<th>( B = 12 )</th>
<th>( \lambda = 6 )</th>
<th>( B = 3 )</th>
<th>( \lambda = 10 )</th>
<th>( B = 2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.95</td>
<td>31</td>
<td>0.284</td>
<td>6.6</td>
<td>0.053</td>
<td>4.0</td>
<td>0.029</td>
</tr>
<tr>
<td>0.85</td>
<td>33.1</td>
<td>0.289</td>
<td>7.4</td>
<td>0.059</td>
<td>4.5</td>
<td>0.033</td>
</tr>
<tr>
<td>0.75</td>
<td>35.4</td>
<td>0.291</td>
<td>8.4</td>
<td>0.067</td>
<td>5.1</td>
<td>0.037</td>
</tr>
<tr>
<td>0.65</td>
<td>37.9</td>
<td>0.288</td>
<td>9.6</td>
<td>0.076</td>
<td>5.8</td>
<td>0.042</td>
</tr>
<tr>
<td>0.55</td>
<td>40.8</td>
<td>0.280</td>
<td>11.2</td>
<td>0.088</td>
<td>6.9</td>
<td>0.050</td>
</tr>
<tr>
<td>0.45</td>
<td>43.8</td>
<td>0.263</td>
<td>13.5</td>
<td>0.105</td>
<td>8.4</td>
<td>0.060</td>
</tr>
<tr>
<td>0.35</td>
<td>47.1</td>
<td>0.234</td>
<td>17.0</td>
<td>0.128</td>
<td>10.6</td>
<td>0.075</td>
</tr>
<tr>
<td>0.25</td>
<td>50.6</td>
<td>0.192</td>
<td>22.5</td>
<td>0.159</td>
<td>14.5</td>
<td>0.100</td>
</tr>
<tr>
<td>0.15</td>
<td>54.3</td>
<td>0.131</td>
<td>32.0</td>
<td>0.191</td>
<td>22.5</td>
<td>0.143</td>
</tr>
</tbody>
</table>

Solidity, \( \sigma \) 0.86 0.088 0.036

Note: \( B \), number of blades; \( c \), airfoil chord length; \( r \), blade section radius; \( R \), rotor radius; \( \lambda \), tip speed ratio; \( \varphi \), angle of relative wind

### 3.9 Generalized Rotor Design Procedure

#### 3.9.1 Rotor design for specific conditions

The previous analysis can be used in a generalized rotor design procedure. The procedure begins with the choice of various rotor parameters and the choice of an airfoil. An initial blade shape is then determined using the optimum blade shape assuming wake rotation. The final blade shape and performance are determined iteratively considering drag, tip losses, and ease of manufacture. The steps in determining a blade design follow.
### 3.9.1.1 Determine basic rotor parameters

1. Begin by deciding what power, $P$, is needed at a particular wind velocity, $U$. Include the effect of a probable $C_P$ and efficiencies, $\eta$, of various other components (e.g., gearbox, generator, pump, etc.). The radius, $R$, of the rotor may be estimated from:

\[
P = C_P \eta \sqrt{\frac{1}{2} \rho R^2 U^3}
\]

2. According to the type of application, choose a tip speed ratio, $\lambda$. For a water pumping windmill, for which greater torque is needed, use $1 < \lambda < 3$. For electric power generation, use $4 < \lambda < 10$. The higher speed machines use less material in the blades and have smaller gearboxes, but require more sophisticated airfoils.

3. Choose a number of blades, $B$, from Table 3.4. Note: If fewer than three blades are selected, there are a number of structural dynamic problems that must be considered in the hub design. One solution is a teetered hub (see Chapter 6).

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>$B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8–24</td>
</tr>
<tr>
<td>2</td>
<td>6–12</td>
</tr>
<tr>
<td>3</td>
<td>3–6</td>
</tr>
<tr>
<td>4</td>
<td>3–4</td>
</tr>
<tr>
<td>$&gt;4$</td>
<td>1–3</td>
</tr>
</tbody>
</table>

4. Select an airfoil. If $\lambda < 3$, curved plates can be used. If $\lambda > 3$, use a more aerodynamic shape.

### 3.9.1.2 Define the blade shape

5. Obtain and examine the empirical curves for the aerodynamic properties of the airfoil at each section (the airfoil may vary from the root to the tip), i.e. $C_l$ vs. $\alpha$, $C_d$ vs. $\alpha$. Choose the design aerodynamic conditions, $C_{l,\text{design}}$ and $\alpha_{\text{design}}$, such that $C_{d,\text{design}}/C_{l,\text{design}}$ is at a minimum for each blade section.

6. Divide the blade into $N$ elements (usually 10–20). Use the optimum rotor theory to estimate the shape of the $i$th blade with a midpoint radius of $r_i$:

\[
\lambda_{r_i} = \lambda (r_i/R)
\]

\[
\varphi_i = (\lambda_{r_i})^{\frac{1}{2}} \tan^{-1}(1/\lambda_{r_i})
\]
\[ c_i = \frac{8\pi r_i}{BCl_{\text{design},i}} (1 - \cos \varphi_i) \]  

(3.9.4)

\[ \theta_{T,i} = \theta_{p,i} - \theta_{p,0} \]  

(3.9.5)

\[ \varphi_i = \theta_{p,i} + \alpha_{\text{design},i} \]  

(3.9.6)

7. Using the optimum blade shape as a guide, select a blade shape that promises to be a good approximation. For ease of fabrication, linear variations of chord, thickness and twist might be chosen. For example, if \( a_i, b_i \) and \( a_2 \) are coefficients for the chosen chord and twist distributions, then the chord and twist can be expressed as:

\[ c_i = a_1 r_i + b_1 \]  

(3.9.7)

\[ \theta_{T,i} = a_2 (R - r_i) \]  

(3.9.8)

3.9.1.3 Calculate rotor performance and modify blade design

8. As outlined above, one of two methods might be chosen to solve the equations for the blade performance.

Method 1 - Solving for \( C_l \) and \( \alpha \) 

Find the actual angle of attack and lift coefficients for the centre of each element, using the following equations and the empirical airfoil curves:

\[ C_{l,i} = 4F_i \sin \varphi_i \frac{(\cos \varphi_i - \lambda_{r,i} \sin \varphi_i)}{\sigma_i \sin \varphi_i + \lambda_{r,i} \cos \varphi_i} \]  

(3.9.9)

\[ \sigma_i' = Bc_i / 2\pi r_i \]  

(3.9.10)

\[ \varphi_i = \alpha_i + \theta_{T,i} + \theta_{p,0} \]  

(3.9.11)

\[ F_i = \frac{(2l \pi) \cos^{-1} \left[ \exp \left( - \frac{(B/2)(1 - (r_i / R))}{(r_i / R) \sin \varphi_i} \right) \right]} \]  

(3.9.12)
The lift coefficient and angle of attack can be found by iteration or graphically. A graphical solution is illustrated in Figure 3.27. The iterative approach requires an initial estimate of the tip loss factor. To find a starting $F_t$, start with an estimate for the angle of the relative wind of:

$$\phi_{i,1} = \left( \frac{\theta}{2} \right) \tan^{-1}\left( \frac{1}{\lambda_{r,i}} \right)$$  \hfill (3.9.13)

For subsequent iterations, find $F_t$ using:

$$\phi_{i,j+1} = \theta_{p,j} + \alpha_{r,i}$$  \hfill (3.9.14)

where $j$ is the number of the iteration. Usually, few iterations are needed.

Finally, calculate the axial induction factor:

$$a_i = \frac{1}{[1 + 4 \sin^2 \phi_i/(\sigma', C_{l,i}, \cos \varphi)]}$$  \hfill (3.9.15)

If $a_i$ is greater than 0.4, use method 2.

![Graphical solution for angle of attack, $\alpha$; $C_l$, two-dimensional lift coefficient; $C_{l,i}$ and $\alpha_i$, $C_l$ and $\alpha$, respectively, for blade section, $i$](image)

**Figure 3.27** Graphical solution for angle of attack, $\alpha$; $C_l$, two-dimensional lift coefficient; $C_{l,i}$ and $\alpha_i$, $C_l$ and $\alpha$, respectively, for blade section, $i$

**Method 2 – Iterative solution for $a$ and $a'$** Iterating to find the axial and angular induction factors using method 2 requires initial guesses for their values. To find initial values, start with values from an adjacent blade section, values from the previous blade design in the iterative rotor design process or use an estimate based on the design values from the starting optimum blade design:

$$\phi_{i,1} = \left( \frac{\theta}{2} \right) \tan^{-1}\left( \frac{1}{\lambda_{r,i}} \right)$$  \hfill (3.9.16)

$$a_{i,1} = \frac{1}{\left[1 + 4 \sin^2 (\phi_{i,1})/\left(\sigma'_{i,design} C_{l,design} \cos \phi_{i,1}\right)\right]}$$  \hfill (3.9.17)
\[ a'_{i,1} = \frac{1-3a_{i,1}}{(4a_{i,1})-1} \]  

(3.9.18)

Having guesses for \( a_{i,1} \) and \( a'_{i,1} \), start the iterative solution procedure for the \( j \)th iteration. For the first iteration \( j = 1 \). Calculate the angle of the relative wind and the tip loss factor:

\[ \tan \varphi_{i,j} = \frac{U(1-a_{i,j})}{\Omega r(1+a'_{i,j})} = \frac{1-a_{i,j}}{(1+a'_{i,j})\lambda_{r,i}} \]  

(3.9.19)

\[ F_{i,j} = (2\pi)\cos^{-1} \left[ \exp \left\{ -\frac{(B/2)[1-(r_{i}/R)]}{(r_{i}/R)\sin \varphi_{i,j}} \right\} \right] \]  

(3.9.20)

Determine \( C_{l,i,j} \) and \( C_{d,i,j} \) from the airfoil lift and drag data, using:

\[ \alpha_{i,j} = \varphi_{i,j} - \theta_{p,i} \]  

(3.9.21)

Calculate the local thrust coefficient:

\[ C_{T_{r,i,j}} = \frac{\sigma_{i}'(1-a_{i,j})^2(C_{l,i,j}\cos \varphi_{i,j} + C_{d,i,j}\sin \varphi_{i,j})}{\sin^2 \varphi_{i,j}} \]  

(3.9.22)

Update \( a \) and \( a' \) for the next iteration. If \( C_{T_{r,i,j}} < 0.96 \):

\[ a_{i,j+1} = \frac{1}{\left[ 1 + \frac{4F_{i,j}\sin^2(\varphi_{i,j})}{\sigma_{i}'C_{l,i,j}\cos \varphi_{i,j}} \right]} \]  

(3.9.23)

If \( C_{T_{r,i,j}} > 0.96 \):

\[ a_{i,j} = \left( \frac{1}{F_{i,j}} \right) \left[ 0.143 + \sqrt{0.0203 - 0.6427(0.889 - C_{T_{r,i,j}})} \right] \]  

(3.9.24)

\[ a'_{i,j+1} = \frac{1}{\frac{4F_{i,j}\cos \varphi_{i,j}}{\sigma_{i}'C_{l,i,j}} - 1} \]  

(3.9.25)
If the newest induction factors are within an acceptable tolerance of the previous guesses, then the other performance parameters can be calculated. If not, then the procedure starts again at Equation 3.9.19 with \( j = j+1 \).

9. Having solved the equations for the performance at each blade element, the power coefficient is determined using a sum approximating the integral in Equation 3.7.12a:

\[
C_p = \sum_{i=1}^{N} \left( \frac{8 \Delta \lambda_r}{\lambda^2} \right) F_i \sin^2 \phi_i (\cos \phi_i - \lambda_r \sin \phi_i) (\sin \phi_i + \lambda_r \cos \phi_i) \left[ 1 - \left( \frac{C_d}{C_l} \right) \cot \phi_i \right] \lambda_r^2
\]

(3.9.26)

If the total length of the hub and blade is assumed to be divided into \( N \) equal length blade elements, then:

\[
\Delta \lambda_r = \lambda_r - \lambda_{r(i-1)} = \lambda/N
\]

(3.9.27)

\[
C_p = \frac{8}{\lambda N} \sum_{i=k}^{N} F_i \sin^2 \phi_i (\cos \phi_i - \lambda_r \sin \phi_i) (\sin \phi_i + \lambda_r \cos \phi_i) \left[ 1 - \left( \frac{C_d}{C_l} \right) \cot \phi_i \right] \lambda_r^2
\]

(3.9.28)

where \( k \) is the index of the first "blade" section consisting of the actual blade airfoil.

10. Modify the design if necessary and repeat steps 8–10, in order to find the best design for the rotor, given the limitations of fabrication.

3.9.2 \( C_p - \lambda \) curves

Once the blade has been designed for optimum operation at a specific design tip speed ratio, the performance of the rotor over all expected tip speed ratios needs to be determined. This can be done using the methods outlined in Section 3.7. For each tip speed ratio, the aerodynamic conditions at each blade section need to be determined. From these, the performance of the total rotor can be determined. The results are usually presented as a graph of power coefficient versus tip speed ratio, called a \( C_p - \lambda \) curve, as shown in Figure 3.28.

\( C_p - \lambda \) curves can be used in wind turbine design to determine the rotor power for any combination of wind and rotor speed. They provide immediate information on the maximum rotor power coefficient and optimum tip speed ratio. Care must be taken in using \( C_p - \lambda \) curves. The data for such a relationship can be found from turbine tests or from modelling. In either case, the results depend on the lift and drag coefficients of the airfoils, which may vary as a function of the flow conditions. Variations in airfoil lift and drag coefficients depend on the airfoil and the Reynolds numbers being considered, but, as shown in Figure 3.10, airfoils can have remarkably different behavior when the Reynolds number changes by as little as a factor of 2.
3.10 Simplified HAWT Rotor Performance Calculation Procedure

Manwell (1990) proposed a simplified method for calculating the performance of a horizontal axis wind turbine rotor that is particularly applicable for an unstalled rotor, but may also be useful under certain stall conditions. The method uses the previously discussed blade element theory and incorporates an analytical method for finding the blade angle of attack. Depending on whether tip losses are included, few or no iterations are required. The method assumes that two conditions apply:

- The airfoil section lift coefficient vs. angle of attack relation must be linear in the region of interest
- The angle of attack must be small enough that the small-angle approximations may be used

These two requirements normally apply if the section is unstalled. They may also apply under certain partially stalled conditions for moderate angles of attack if the lift curve can be linearized.

The simplified method is the same as method 1 outlined above, with the exception of a simplification for determining the angle of attack and the lift coefficient for each blade section. The essence of the simplified method is the use of an analytical (closed-form) expression for finding the angle of attack of the relative wind at each blade element. It is assumed that the lift and drag curves can be approximated by:

\[ C_l = C_{l,0} + C_{l,\alpha} \alpha \quad (3.10.1) \]

\[ C_d = C_{d,0} + C_{d,\alpha1} \alpha + C_{d,\alpha2} \alpha^2 \quad (3.10.2) \]
When the lift curve is linear and when small-angle approximations can be used, it can be shown that the angle of attack is given by:

$$\alpha = \sqrt{\frac{q_2^3 - 4q_1q_3 - q_2}{2q_3}}$$  \hspace{1cm} (3.10.3)$$

where:

$$q_1 = C_{l,0} d_2 - \frac{4F}{\sigma'} d_1 \sin \theta_p$$ \hspace{1cm} (3.10.4)$$

$$q_2 = C_{l,a} d_2 + d_1 C_{l,0} - \frac{4F}{\sigma'} (d_1 \cos \theta_p - d_2 \sin \theta_p)$$ \hspace{1cm} (3.10.5)$$

$$q_3 = C_{l,a} d_1 + \frac{4F}{\sigma'} d_2 \cos \theta_p$$ \hspace{1cm} (3.10.6)$$

$$d_1 = \cos \theta_p - \lambda_r \sin \theta_p$$ \hspace{1cm} (3.10.7)$$

$$d_2 = \sin \theta_p + \lambda_r \cos \theta_p$$ \hspace{1cm} (3.10.8)$$

Using this approach, the angle of attack can be calculated from Equation 3.10.3 once an initial estimate for the tip loss factor is determined. The lift and drag coefficients can then be calculated from Equation 3.10.1 and 3.10.2, using Equation 3.9.14. Iteration with a new estimate of the tip loss factor may be required.

The simplified method provides angles of attack very close to those of the more detailed method for many operating conditions. For example, results for the analysis of one blade of the University of Massachusetts WF-1 wind turbine are shown in Figure 3.29. This was a three-bladed turbine with a 10 m rotor, using near optimum tapered and twisted blades. The lift curve of the NACA 4415 airfoil was approximated by $C_l = 0.368 + 0.0942 \alpha$. The drag coefficient equation constants were 0.00994, 0.000259, and 0.0001055. Figure 3.29 compares the results from the simplified method and the conventional strip theory method for the angle of attack for one of the blade elements. The point at which the curves cross the empirical lift line determines the angle of attack and the lift coefficient. Also plotted on Figure 3.29 is the axial induction factor, $a$, for the section. Note that it is the right-hand intersection point which gives a value of $a < 1/2$, as is normally the case.
3.11 Effect of Drag and Blade Number on Optimum Performance

At the beginning of the chapter, the maximum theoretically possible power coefficient for wind turbines was determined as a function of tip speed ratio. As explained in this chapter, airfoil drag and tip losses that are a function of the total number of blades reduce the power coefficients of wind turbines. The maximum achievable power coefficient for turbines with an optimum blade shape but a finite number of blades and aerodynamic drag has been calculated by Wilson et al. (1976). Their fit to the data is accurate to within 0.5% for tip speed ratios from 4 to 20, lift to drag ratios \((C_l/C_d)\) from 25 to infinity and from one to three blades \((B)\):

\[
C_{p,\text{max}} = \left( \frac{16}{27} \right) \lambda \left[ \frac{1.32 + \left( \frac{\lambda - 8}{20} \right)^2}{\lambda + \frac{1}{2B}} \right]^{-1} - \left( \frac{0.57}{C_l/C_d} \right) \lambda^2 \left( \lambda + \frac{1}{2B} \right)
\]  

(3.11.1)

Figure 3.30, based on this equation, shows the maximum achievable power coefficients for a turbine with 1, 2, and 3 optimum blades and no drag. The performance for ideal conditions (an infinite number of blades) is also shown. It can be seen that the fewer the blades the lower the possible \(C_p\) at the same tip speed ratio. Most wind turbines use two or three blades and, in general, most two-bladed wind turbines use a higher tip speed ratio than most three-bladed wind turbines. Thus, there is little practical difference in the maximum achievable \(C_p\) between typical two- and three-bladed designs, assuming no drag. The effect of the lift to drag ratio on maximum achievable power coefficients for a three-bladed rotor is shown in Figure 3.31. There is clearly a significant reduction in maximum achievable power as the airfoil drag increases. For reference, the DU-93-W-210 airfoil has a maximum
$C_l/C_d$ ratio of 140 at an angle of attack of 6 degrees, and the 19\% thick LS(1) airfoil has a maximum $C_l/C_d$ ratio of 85 at an angle of attack of 4 degrees. It can be seen that it clearly benefits the blade designer to use airfoils with high lift to drag ratios. Practical rotor power coefficients may be further reduced as a result of non-optimum blade designs that are easier to manufacture, the lack of airfoils at the hub and aerodynamic losses at the hub end of the blade.

![Figure 3.30](image-url) Maximum achievable power coefficients as a function of number of blades, no drag

![Figure 3.31](image-url) Maximum achievable power coefficients of a three-bladed optimum rotor as a function of the lift to drag ratio, $C_l/C_d$. 

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It can be seen that it clearly benefits the blade designer to use airfoils with high lift to drag ratios. Practical rotor power coefficients may be further reduced as a result of non-optimum blade designs that are easier to manufacture, the lack of airfoils at the hub and aerodynamic losses at the hub end of the blade.

![Figure 3.30](image-url) Maximum achievable power coefficients as a function of number of blades, no drag

![Figure 3.31](image-url) Maximum achievable power coefficients of a three-bladed optimum rotor as a function of the lift to drag ratio, $C_l/C_d$. 

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Aerodynamics of Wind Turbines
3.12  Advanced Aerodynamic Topics

As mentioned in the beginning of this chapter, the aerodynamic performance of wind turbines are primarily a function of the steady state aerodynamics that are discussed above. The analysis presented in this chapter provides a method for determining average loads on a wind turbine. There are, however, a number of important steady state and dynamic effects that cause increased loads or decreased power production from those expected with the BEM theory presented here, especially increased transient loads. An overview of those effects is provided in this section, including non-ideal steady state effects, the influence of turbine wakes and unsteady aerodynamics. This section also includes comments on computer programs that can be used for rotor performance modelling and approaches to modelling rotor aerodynamics other than BEM methods.

3.12.1  Non-ideal steady state aerodynamic issues

Steady state effects that influence wind turbine behavior include the degradation of blade performance due to surface roughness, the effects on blade performance of stall and blade rotation.

As mentioned above in Section 3.4.5, blade surfaces roughened by damage and debris can significantly increase drag and decrease the lift of an airfoil. This has been shown to decrease power production by as much as 40% on certain airfoils. The only solution is frequent blade repair and cleaning or the use of airfoils that are less sensitive to surface roughness.

Parts of a wind turbine blade may operate at times in the stall region. On stall-controlled horizontal axis rotors much of the blade may be stalled under some conditions. Stalled airfoils do not always exhibit the simple relationship between the angle of attack and aerodynamic forces that are evident in lift and drag coefficient data. The turbulent separated flow occurring during stall can induce rapidly fluctuating flow conditions and rapidly fluctuating loads on the wind turbine.

Finally, the lift and drag behavior of airfoils are measured in wind tunnels under non-rotating conditions. Investigation has shown that the same airfoils, when used on a horizontal axis wind turbine, may exhibit delayed stall and may produce more power than expected. The resulting unexpectedly high loads at high wind speeds can reduce turbine life. This behavior has been linked to spanwise pressure gradients that result in a spanwise velocity component along the blade that helps keep the flow attached to the blade, delaying stall and increasing lift.

3.12.2  Turbine wakes

Many of the primary features of the air flow in and around wind turbines are described by the results of BEM theory: the induced velocities due to power production and the rotation of the turbine wake and the expanding wake downwind of the turbine. The actual flow field, however, is much more complicated. Some of the details of flow around and behind a horizontal axis wind turbine are described here. The consequences of these flow patterns affect the wind turbine and may result in a skewed wake, which causes increased...
fluctuating loads which are not predicted by BEM theory. Before considering their consequences, the details of turbine wakes are described in the following paragraph.

Wind turbine wakes are often thought of as consisting of a near wake and a far wake (Voutsinas et al., 1993). The difference between the near and far wakes is a function of the spatial distribution and intensity of turbulence in the flow field. Fluid flow modelling and experiments have shown that each blade generates a sheet of vortices that is transported through the wake by the mean axial and rotational flow in the wake. In addition to the vortex sheet from the trailing edge of the blades, vortices generated at the hub and especially strong vortices, generated at the blade tips, are also convected downstream. The tip vortices cause the tip losses mentioned in Section 3.5. All of these vortices and mechanically generated turbulence are dissipated and mixed in the near wake (within 1 to 3 rotor diameters downstream of the rotor). The stream of tip vortices from each blade merge into the near wake to form a cylinder sheet of rotating turbulence as they mix and diffuse through the flow (Sorensen and Shen, 1999). Much of the periodic nature of the flow is lost in the near wake (Ebert and Wood, 1994). Thus, turbulence and vorticity generated at the rotor are diffused in the near wake, resulting in more evenly distributed turbulence and velocity profiles in the far wake. Meanwhile, the mixing of the slower axial flow of the turbine wake with the free stream flow slowly re-energizes the flow. The vortex sheet from the tip vortices results in an annular area in the far wake of higher relative turbulence surrounding the less turbulent core of the wake. Mixing and diffusion continue in the far wake until the turbine-generated turbulence and velocity deficit with respect to the free stream flow have disappeared.

The consequences of the vorticity and turbulence in turbine wakes are increased loads and fatigue. The most obvious effect is the increased turbulence in the flow at turbines that are downwind of other turbines in a wind farm (see Chapter 8). The nature of turbine wakes also affects the loads on the turbine producing the wake. For example, tip and hub vortices reduce the energy capture from the rotor.

Another important effect occurs with off-axis winds, i.e. those whose direction is not perpendicular to the plane of the rotor. Off-axis flows, whether due to yaw error or vertical wind components, result in a skewed wake in which the wake is not symmetric with the turbine axis. Skewed wakes result in the downwind side of the rotor being closer to the wake centerline than the upwind side of the rotor. The result is higher induced velocities on the downwind side of the rotor than the upwind side. This effect has been shown to result in higher turbine forces than otherwise would be expected (Hansen, 1992). One commonly used approach to modelling the effects of a skewed wake is the Pitt and Peters model (Pitt and Peters, 1981; Goankar and Peters, 1986). The model applies a multiplicative correction factor to the axial induction factor that is a function of yaw angle, radial position and blade azimuth angle. For more information on its use in wind turbine modelling, see Hansen (1992).

3.12.3 Unsteady aerodynamic effects

There are a number of unsteady aerodynamic phenomena that have a large effect on wind turbine operation. The turbulent eddies carried along with the mean wind cause rapid changes in speed and direction over the rotor disk. These changes cause fluctuating
aerodynamic forces, increased peak forces, blade vibrations, and significant material fatigue. Additionally, the transient effects of tower shadow, dynamic stall, dynamic inflow and rotational sampling (all explained below) change turbine operation in unexpected ways. Many of these effects occur at the rotational frequency of the rotor or at multiples of that frequency. Effects that occur once per revolution are often referred to as having a frequency of \(1P\). Similarly, effects that occur at 3 or \(n\) times per revolution of the rotor are referred to as occurring at a frequency of \(3P\) or \(nP\).

**Tower shadow** refers to the wind speed deficit behind a tower caused by the tower obstruction. The blades of a downwind rotor with \(B\) blades will encounter the tower shadow once per revolution, causing a rapid drop in power and \(BP\) vibrations in the turbine structure.

**Dynamic stall** refers to rapid aerodynamic changes that may bring about or delay stall behavior. Rapid changes in wind speed (for example, when the blades pass through the tower shadow) cause a sudden detachment and then reattachment of airflow along the airfoil. Such effects at the blade surface cannot be predicted with steady state aerodynamics, but may affect turbine operation, not only when the blades encounter tower shadow, but also during operation in turbulent wind conditions. Dynamic stall effects occur on time scales of the order of the time for the relative wind at the blade to traverse the blade chord, approximately \(c/\Omega r\). For large wind turbines, this might be on the order of 0.2 seconds at the blade root to 0.01 seconds at the blade tip (Snel and Schepers, 1991). Dynamic stall can result in high transient forces as the wind speed increases, but stall is delayed. A variety of dynamic stall models have been used in computer rotor performance codes including those of Gormont (1973) and Beddoes (Björck et al., 1999). The Gormont model, for example, modifies the angle of attack calculated with the BEM theory by adding a factor that depends on the rate of change of the angle of attack.

**Dynamic inflow** refers to the response of the larger flow field to turbulence and changes in rotor operation (pitch or rotor speed changes, for example). Steady state aerodynamics suggests that increased wind speed and, thus, increased power production should result in an instantaneous increase in the axial induction factor and changes in the flow field upstream and downstream of the rotor. During rapid changes in the flow and rapid changes in rotor operation, the larger field cannot respond quickly enough to instantly establish steady state conditions. Thus the aerodynamic conditions at the rotor are not necessarily the expected conditions, but an ever-changing approximation as the flow field changes. The time scale of dynamic flow effects is on the order of \(D/U\), the ratio of the rotor diameter to the mean ambient flow velocity. This might be as much as 10 seconds (Snel and Schepers, 1991).

Phenomena occurring slower than this can be considered using a steady state analysis. For more information on dynamic inflow see Snel and Schepers (1991, 1993) and Pitt and Peters (1981).

Finally, **rotational sampling** (see Connell, 1982) causes some unsteady aerodynamic effects and increases fluctuating loads on the wind turbine. These effects are all induced or complicated because the wind as seen by the rotor is constantly changing as the rotor rotates. The general flow turbulence may bring wind speed changes on a time scale of about 5 seconds. The turbulent eddies may be smaller than the rotor disk, resulting in different winds at different parts of the disk. If the blades are rotating once a second, the blades 'sample' different parts of the flow field at a much faster rate than the general changes in the wind field itself, causing rapidly changing flow at the blade.
3.12.4 Computer codes for performance and load estimation

A number of computer codes are available that can predict rotor performance and aerodynamic loads. The National Renewable Energy Laboratory (NREL) in Boulder, CO, has supported the development of a number of these codes and has made some of the codes available over the Internet. The aerodynamic performance codes that are available from NREL include:

- WT_Perf
- YawDyn and AeroDyn

**WT_Perf**: WT_Perf (Buhl, 2000) is a rotor performance code for horizontal axis wind turbines. The aerodynamics analysis uses momentum and strip theory to determine blade performance, including corrections for tip losses and wind shear. Three-dimensional aerodynamics calculations have also been added. The code and a users manual are available from NREL.

**YawDyn**: YawDyn (Hansen, 1992) is a complete aerodynamics and dynamics analysis code for constant speed horizontal axis wind turbines. The code was designed to evaluate wind turbine yaw dynamics. The aerodynamics part of the code, AeroDyn, uses a detailed implementation of strip theory with modifications to improve results for unsteady winds. More details on YawDyn are given in Section 4.4.1.

3.12.5 Other performance prediction and design methods

In this chapter, the BEM theory approach has been used to predict rotor performance. An iterative approach to blade design has also been outlined based on the analysis methods detailed in the text. There are other approaches to predicting blade performance and to designing blades that may be more applicable in some situations. Some of the disadvantages of the BEM theory include errors under conditions with large induced velocities (Glauert, 1948) or yawed flow and inability to predict delayed stall due to rotational effects.

Vortex wake methods have been used in the helicopter industry in addition to BEM methods. Vortex wake methods calculate the induced velocity field by determining the distribution of vorticity in the wake. This method is computationally intensive, but promises to have advantages for yawed flow and operation subject to three-dimensional boundary layer effects (Hansen and Butterfield, 1993).

There are also other possible theoretical approaches. Researchers at Delft University of Technology have reported initial work on a model employing asymptotic acceleration potential methods (Hansen and Butterfield, 1993). Cascade Theory, often used in turbomachinery design, has also been used to analyse wind turbine performance. Cascade theory takes into consideration aerodynamic interactions between blades. Although it is more computationally intensive, cascade theory has been shown to provide better results than BEM theory for high-solidity, low tip-speed rotors (Islam and Islam, 1994). Computational fluid dynamics (CFD), while more computationally intensive, has also been applied to wind turbine rotors (see, for example, Sorenson and Michelsen, 2002, and Duque et al., 1999).
Finally, each of these analysis methods could be used in an iterative fashion to define the final blade design for a wind turbine, but work has been also preformed to develop a computer code to approach the blade design problem from the opposite direction (Selig and Tangler, 1992). This approach allows the designer to input desired rotor performance characteristics and blade aerodynamic characteristics and the code determines the corresponding blade geometry. The code has been used successfully to design blades for a commercial wind turbine.

References


