Wind Energy:
Wind Turbine Geometry

MECE E4211 – Energy Sources and Conversion
Week 9

Supplemental Reading:
Recap: Motivation for Wind Power

- After solar energy, wind is the most abundant renewable power source.
- High availability but variable generation
Drag Wind Power

- Traditional drag devices are relatively inefficient but are demonstrative of why other wind technology is in use.
- **Note**: Blade velocity in this context is $v$, gas velocity is $u$ (opposite of gas turbine)
- **Power** = Drag $\cdot$ Velocity
  - $D = C_D \left[ \frac{1}{2} \rho (u - v)^2 c \ell \right]$
  - $P = Dv$
    - $= C_D \left[ \frac{1}{2} \rho (u - v)^2 c \ell \right] v$
    - $= \frac{1}{2} \rho u^3 C_D (1 - v/u)^2 c \ell (v/u)$
    - $= \frac{1}{2} \rho u^3 A_p C_D (1 - v/u)^2 (v/u)$
  - $P = 0$ at $v = 0$ or $v = u$
  - $P_{max}$ occurs at $v/u = \frac{1}{3}$ (and $C_p = \frac{4}{27} C_D$)

$C_D = 1.4$ for this shape

Cross sectional area, $A_p = c \ell$
Lift-type Wind Turbines

- Wind is **parallel** to axis of rotation and perpendicular to motion of blades
- Assume velocity of blade, $v$, is constant (short distances)
- The angle and size of the blade relative to the wind may vary from the hub to the tip of the turbine.
Wind Velocity Triangle

• The wind relative to the blade is $v_r$.
• The value for $\theta$ is a property specific to each system.
  – Close to 0° at the tip, increased from hub to tip
• $v_r$ and $\beta$ will constantly change due to shifting winds, blade speed.
• Blade speed $v$ changes from hub to tip (proportional to $r$)
Wind Velocity Triangle

- Increasing angle of attack $\alpha$ will increase lift until a certain point, where it drops due to stall.

- $v_r$ and $\beta$ will constantly change due to shifting winds, blade speed.
Angle of Attack
Angle of Attack

Blade at low, medium & high angles of attack
Angle of Attack

Apparent wind angles
Force Triangle

- \( L = c \cdot \ell \cdot \frac{1}{2} \rho v_r^2 \cdot C_L \)
- \( D = c \cdot \ell \cdot \frac{1}{2} \rho v_r^2 \cdot C_D \)
  - \( \ell \) is a length in the radial direction
- \( F = L \cdot \sin \beta - D \cdot \cos \beta \)
  - Net force is in the direction of blade motion.
Calculation of Power

Power can be calculated in terms of only \( u \) and \( v \).

- \( P = F \cdot v \)
- \( = (L \cdot \sin \beta - D \cdot \cos \beta) \cdot v \)
- \( = c \cdot l \cdot \frac{1}{2} \rho \cdot v_r^2 \cdot v \cdot (C_L \cdot \sin \beta - C_D \cdot \cos \beta) \)
- \( = A_p \cdot \frac{1}{2} \rho \cdot (v_r^2/u^2) \cdot (v/u) \cdot u^3 \cdot \sin \beta \cdot (C_L - C_D / \tan \beta) \)
- \( = \frac{1}{2} \rho \cdot A_p \cdot (1/\sin^2 \beta) \cdot (v/u) \cdot u^3 \sin \beta \cdot (C_L - C_D / \tan \beta) \)

\[
\frac{P}{\frac{1}{2} \rho u^3 A_p} = \frac{v}{u} \sqrt{1 + \left(\frac{v}{u}\right)^2 \left(C_L - \frac{v}{u} C_D\right)}
\]

Since \( \sin \beta = u/v_r \) and \( \tan \beta = u/v \):
Coefficient of Power

- Equation for power can be simplified if \( v \) (turbine velocity) is much larger than \( u \) (wind velocity).

\[
\frac{P}{\frac{1}{2} \rho u^3 A_p} = \frac{v}{u} \sqrt{1 + \left(\frac{v}{u}\right)^2 \left(C_L - \left(\frac{v}{u}\right) C_D\right)} \quad \text{\( \sqrt{1 + \left(\frac{v}{u}\right)^2 \approx \frac{v}{u} \) if \( \left(\frac{v}{u}\right)^2 \gg 1 \)}
\]

\[
\frac{P}{\frac{1}{2} \rho u^3 A_p} = \left(\frac{v}{u}\right)^2 \left(C_L - \left(\frac{v}{u}\right) C_D\right) = C_P
\]

\( C_P = \text{Coefficient of Power (dimensionless)} \)
Coefficient of Power

- Maximum value of Cp can be calculated by finding local maximum with respect to v/u.

\[ C_P = \left( \frac{v}{u} \right)^2 \left( C_L - \left( \frac{v}{u} \right) C_D \right) \Rightarrow \frac{d(C_P)}{d\left( \frac{v}{u} \right)} = 2 \left( \frac{v}{u} \right) C_L - 3 \left( \frac{v}{u} \right)^2 C_D \rightarrow \text{Max at } \frac{v}{u} = \frac{2C_L}{3C_D} \]

\[ C_P \bigg|_{\frac{v}{u} = \frac{2C_L}{3C_D}} = \left( \frac{2C_L}{3C_D} \right)^2 \left( C_L - \left( \frac{2C_L}{3C_D} \right) C_D \right) \rightarrow C_{P,\text{maximum}} = \frac{4}{27} C_L \left( \frac{C_L}{C_D} \right)^2 \]
Calculation of Power

\[
\frac{P}{\frac{1}{2} \rho u^3 A_p} = \left(\frac{v}{u}\right)^2 \left(c_L - \left(\frac{v}{u}\right) c_D\right) = C_P
\]

- \( A_p \), the cross-sectional or “plan form” area, is the total solid blade area, \( n \cdot R \cdot C_{ave} \)
  where \( C_{ave} \) is computed to ensure that actual blade area seen from the axial direction is equal to \( R \cdot C_{ave} \).
Calculation of Torque

• Torque exerted on the turbine blade will vary across its length, due to increases in blade velocity closer to its tips.
  – Recall $F = L \cdot \sin \beta - D \cdot \cos \beta$ and $v/u = \tan \beta$
  – As $r$ increases, $v$ increases but $u$ does not.
  – **Torque** $(T) = F \cdot r$ and will therefore vary with $r$.

• **Ideally, torque should be constant along the blade, to minimize internal bending.**
Calculation of Torque

\[ T = \frac{1}{2} \rho u^2 dA_p R \frac{r}{R} \left( \sqrt{1 + \left( \frac{v}{u} \right)^2} C_L \left( 1 - \left( \frac{v}{u} \right) \frac{C_D}{C_L} \right) \right) \]

- Drag/lift are functions of r, assuming blade shape drag/lift characteristics is constant.

- Variables that may be altered:
  - \( \theta \), blade tilt (positive values only)
  - C, Chord length
Approximations for $C_{P,\text{max}}$

Equations such as the following [Wilson et al. 1976]:

$$C_{P,\text{max}} = 0.593 \left[ \frac{\lambda B^{0.67}}{1.48 + (B^{0.67} - 0.04)\lambda + 0.0025\lambda^2} - \frac{1.92\lambda^2 B}{1 + 2\lambda B} \frac{D/L}{\lambda} \right]$$  \hspace{1cm} (5-40)

Where $D/L = \text{ratio of } C_p \text{ to } C_r \text{ at the design angle of attack; drag-to-lift ratio}$

Figure 5-17 illustrates the application of Equation 5-40. The Glauert ideal HAWT performance ($B \to \infty$ and $D/L = 0$; data from Table 5-2) forms an upper bound.

- Empirical results: a function of $B$ (number of blades), $D/L$, and tip-speed ratio, $\lambda$, at designed wind speed.
Wind Characteristics

- The relative wind speed \((u/c)\) at a given time follows the Rayleigh distribution:
  \[
  (u) = \frac{k}{c} \left(\frac{u}{c}\right)^{k-1} \exp\left(\frac{u}{c}\right)^k
  \]
  - \(c\) is scale parameter

- **Shape factor \((k)\) describes variance**
  - \(k = 2\) common for wind

Figure 8-7. Wind speed frequency distributions in dimensionless form, based on the Weibull duration curves. Annual average wind speeds are indicated.
Rayleigh Averages

- Mean wind speed can be calculated using a gamma function.
  \[ \bar{u} = c \Gamma \left( 1 + \frac{1}{k} \right) \]

- Power is proportional to the cube of wind speed.
  – Note: Average of cubes is not the same as average of cubes.
  \[ \bar{u}^n = c^n \Gamma \left( 1 + \frac{n}{k} \right) \]
  \[ \bar{u}^3 = c^3 \Gamma \left( 1 + \frac{3}{k} \right) = c^3 (1.33) \]
Power Averages

• Power Density (W/m²)

\[ E = \frac{1}{2} \rho \bar{u}^3 = \frac{1}{2} \rho (c^3 \cdot 1.33) \]

• For \( k = 2 \), \( \bar{u} = c \cdot \Gamma(1.5) = 0.89 \)

\[ E = \frac{1}{2} \rho \left( \frac{\bar{u}}{0.89} \right)^3 \cdot 1.33 = \frac{1}{2} \rho \bar{u}^3 \cdot 1.886 \]

• Therefore: if average windspeed increases by 15%, annual energy will increase by roughly 50%!

\[ 1.15^3 \approx 1.5 \]
Meteorological Data

![Graph showing wind speed and hours/year](image)
Performance Data

39 m Diameter, 500 kW Nameplate Capacity Turbine
Electrical Output

Annual Energy Output for 39-m 500-kW Wind Turbine Using Power Curve

<table>
<thead>
<tr>
<th>Wind Speed Bin (m/s)</th>
<th>Frequency of Occurrence</th>
<th>Capacity kW</th>
<th>Energy kWh</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>%</td>
<td>hr/yr</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.0992</td>
<td>869</td>
<td>7</td>
</tr>
<tr>
<td>5</td>
<td>0.1074</td>
<td>941</td>
<td>29</td>
</tr>
<tr>
<td>6</td>
<td>0.1080</td>
<td>946</td>
<td>63</td>
</tr>
<tr>
<td>7</td>
<td>0.1023</td>
<td>896</td>
<td>107</td>
</tr>
<tr>
<td>8</td>
<td>0.0919</td>
<td>805</td>
<td>161</td>
</tr>
<tr>
<td>9</td>
<td>0.0788</td>
<td>690</td>
<td>224</td>
</tr>
<tr>
<td>10</td>
<td>0.0645</td>
<td>565</td>
<td>293</td>
</tr>
<tr>
<td>11</td>
<td>0.0507</td>
<td>444</td>
<td>362</td>
</tr>
<tr>
<td>12</td>
<td>0.0383</td>
<td>335</td>
<td>423</td>
</tr>
<tr>
<td>13</td>
<td>0.0278</td>
<td>243</td>
<td>467</td>
</tr>
<tr>
<td>14</td>
<td>0.0194</td>
<td>170</td>
<td>489</td>
</tr>
<tr>
<td>15</td>
<td>0.0131</td>
<td>114</td>
<td>497</td>
</tr>
<tr>
<td>16</td>
<td>0.0085</td>
<td>74</td>
<td>500</td>
</tr>
<tr>
<td>17</td>
<td>0.0083</td>
<td>46</td>
<td>500</td>
</tr>
<tr>
<td>18</td>
<td>0.0032</td>
<td>28</td>
<td>500</td>
</tr>
<tr>
<td>19</td>
<td>0.0019</td>
<td>16</td>
<td>500</td>
</tr>
<tr>
<td>20</td>
<td>0.0011</td>
<td>9</td>
<td>500</td>
</tr>
<tr>
<td>21</td>
<td>0.0006</td>
<td>5</td>
<td>500</td>
</tr>
<tr>
<td>22</td>
<td>0.0003</td>
<td>3</td>
<td>500</td>
</tr>
<tr>
<td>23</td>
<td>0.0002</td>
<td>1</td>
<td>500</td>
</tr>
<tr>
<td>24</td>
<td>0.0001</td>
<td>1</td>
<td>500</td>
</tr>
<tr>
<td>25</td>
<td>0.0000</td>
<td>0</td>
<td>500</td>
</tr>
<tr>
<td>26</td>
<td>0.0000</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Annual energy output = 1,287,200 kWh