Consider a solar module with 36 cells. Each cell is 100cm$^2$ in area, with a reverse saturation current of $1 \times 10^{-9}$ mA/cm$^2$. Under full sunlight, each cell has a short-circuit current of 45 mA/cm$^2$ at 25°C. The cells are connected in series with one another within the module.

Recall that in an ideal situation, the cells of this solar module can be thought of as a current source driven by sunlight in conjunction with a diode. In this case, these cells have an $I$-$V$ curve that can be represented as:

$$I = I_{SC} - I_d = I_{SC} - I_0\left(e^{qV/k_BT} - 1\right)$$

where $q$ is the elementary charge ($1.602 \times 10^{-19}$ C), $k_B$ is the Boltzmann constant ($1.3806 \times 10^{-23}$ m$^2$-kg/K-s$^2$), and $T$ is ambient temperature (K).

1.) What is the total open circuit voltage of a given cell within this module, under ideal conditions?

2.) Plot a cell’s $I$-$V$ curve and power-versus-voltage curve in the programming language of your choice (e.g. MATLAB, R, etc.) What is the maximum power point (MPP) and fill factor, under full sunlight conditions?

Real photovoltaic systems often have inherent losses that result in deviations from these values, and need to be accounted for to properly represent a given system’s behavior in non-ideal situations. Adding a factor for parallel leakage resistance, $R_p$, the current of the cell becomes:

$$I = I_{SC} - I_d - \frac{V}{R_p} = I_{SC} - I_0\left(e^{qV/k_BT} - 1\right) - \frac{V}{R_p}$$

3.) Assume $R_p = 1.2\Omega$. Plot the new $I$-$V$ curve and power-versus-voltage curve. What is the maximum power point (MPP) and fill factor, under full sunlight conditions? How much power loss is experienced due to parallel resistance?

4.) What parallel resistance would be necessary to experiences losses of less than 1%?

Adding a series resistance factor, $R_s$, representative of contact resistances and inter-cell connections, yields voltage deviations proportional to this term:
\[ I = I_{SC} - I_d - \frac{(V + I \cdot R_s)}{R_p} = I_{SC} - I_0 \left( e^{\frac{q(V + I \cdot R_s)}{kT}} - 1 \right) - \frac{(V + I \cdot R_s)}{R_p} \]

5.) Assume \( R_s = 50 \text{m}\Omega \). Plot this I-V curve and power-versus-voltage curve. What is the maximum power point (MPP) and fill factor, under full sunlight conditions? How much power loss is experienced due to series resistance? (Note: The current is now present on both sides of this equation, but it can be more easily manipulated by creating a new variable for voltage with and without the series deviation and performing appropriate substitutions.)

6.) What resistance would be necessary to experiences series losses of less than 1%?

7.) Using the \( R_p \) and \( R_s \) values given, what is the total power output of the entire solar module in full sunlight?

8.) Consider a PV array that is connected to a simple water heater. This heater that needs to be able to deliver as much as 2kW of power at 40V DC. Accounting for resistive losses, how many PV modules are necessary to deliver the desired load? What configuration of modules is necessary?

The power output of a photovoltaic module can be severely affected when even a small fraction of its surface is shaded. Under such conditions, the current source and diode elements of the photovoltaic cell equivalent circuit no longer contribute to system voltage. Using the idealized equation, this would have meant that no current is delivered through the shaded cell as the circuit is essentially open. However, accounting for resistive losses, we are left with the following module voltage with a single shaded cell:

\[ V_{SH} = \left( \frac{n-1}{n} \right) V - I(R_p + R_s) \]

9.) Using the same corrected system from Problem #5, plot the I-V curves and power output curves for a module that has 1, 3, and 5 modules shaded. Compared to the ideal case in Problem #2, how much power loss is experienced for these cases?

10.) Assume that current is maintained and that all resistive power loss will transfer to heat. For each of the shaded cases, if each solar cell is roughly \(~150\text{g}\) and has a heat capacity of \(750\text{J/kg-K}\), roughly what temperature will the shaded cell(s) elevate to after one hour of operation under these conditions?