Solar Energy I:
Introduction and Solar Geometry

MECE E4211 – Energy Sources and Conversion
Week 10/11

Dominant Solar Technology

- Photovoltaic (PV) cells
- Solar-thermal systems
- Concentrated Photovoltaic (CPV) systems
Historically, module cost has dropped 20% for every doubling of capacity.

- Increasing production has led to lower per-module costs, feasibility of implementation.
Methods of Heat Transfer

• Conduction
  – Fourier’s Law
  – Flux: \( \dot{q} = -k \frac{dT}{dx} = -k \nabla T \) [\( = \) \( W/m^2 \)]

  
  \[
  \frac{dT}{dx} = -k \nabla T \\
  q = W/m^2
  \]

  Thermal conductivity (material dependent)

• Convection
  – Newton’s Law of Cooling
  – Flux: \( \dot{q} = h(T_{wall} - T_{bulk}) \) [\( = \) \( W/m^2 \)]

  Convective heat transfer coefficient (material, flow dependent)

• Thermal Radiation
  – Stefan-Boltzmann Law for Blackbody Radiation
  – Flux: \( \dot{q} = \sigma F \varepsilon (T_{hot}^4 - T_{cold}^4) \)

  Stefan-Boltzmann Constant:
  \( \sigma = 5.67 \times 10^{-8} \) [\( = \) \( W/m^2K^4 \)]

  View factor (\( F = 1 \) for parallel plates)

  Emissivity of surfaces
Thermal Radiation and Planck’s Law

- Solar energy transmitted via radiation over range of \( \lambda \)
- Radiative energy is not dependent on material, but is a strong function of temperature.

\[
\dot{Q} = \int_0^\infty Q(\lambda) d\lambda = A\sigma T^4 \quad [=] \quad W
\]

**Wien’s Displacement Law:**

\[
\lambda_{\text{max}} T = 2898 \ \mu m-K
\]
The sun’s energy distribution spectrum (yellow) roughly aligns with a blackbody spectrum of 5250°C (black line). **Travelling through the atmosphere results in absorptive losses** and less irradiance at the surface (red).
Atmospheric Radiative Balance

EARTH'S ENERGY BUDGET

Incoming solar energy 100%

Reflected by atmosphere 6%
Reflected by clouds 20%
Reflected from earth's surface 4%

Radiated to space from clouds and atmosphere 64%
Radiated directly to space from earth 6%

Absorbed by atmosphere 16%

Absorbed by clouds 3%
Conduction and rising air 7%

Carried to clouds and atmosphere by latent heat in water vapor 23%

Absorbed by land and oceans 51%

Source: http://marine.rutgers.edu/mrs/education/class/yuri/erb.html
Atmospheric Radiative Balance

Absorbance can occur in both the downgoing (radiation from the sun) and upgoing (radiation from the Earth) directions.

Relevant atmospheric gases vary between the two directions due to the different temperature and wavelength regimes in which they are operating.

Source: Robert A. Rohde/Global Warming Art.
Air Mass (AM) Standards

- Solar spectra are named by their traversed air mass - depending on the angle of the sun (via time/location), the amount of atmospheric gases that must be traversed before reaching the surface will vary.

\[
AM = \frac{L}{L_0} \approx \frac{1}{\cos Z}
\]

Zenith angle = 48.1°, Sea level
AM = 1 / cos(48.1) = 1.5

L_0 = Optical path length
L_0 = Optical path length at zenith angle 0°, sea level

Directly normal to surface, Sea level: AM = 1
Air Mass (AM) Standards

- **AM 0** - Averaged measured data from *satellites, space shuttles, high-altitude aircraft*, etc.

- **AM1.5** – *Surface level, factors atmospheric gases/aerosols, humidity, albedo, etc.* at 48.1° solar zenith angle.
  - **AM1.5 Direct + Circumsolar**: Solar radiation from the sun and the cone of sky 2.5° around sun, direct panel exposure. (ASTM G173-03)
  - **AM1.5G (Global)**: Solar radiation from sun, entire sky, surface albedo effects, panel angle 37° (incidence angle of 11.1°) (ASTM G173-03)
  - **AM1.5G is used to calibrate solar cells** and gives higher cell efficiencies than AM0, and higher power per sq. meter than AM1.5 Direct + Circumsolar. (ASTM E490-00a)

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**AM1.5 Direct + Circumsolar**

48.1°

Earth

**AM1.5 Global**

11.1°

Earth
Air Mass (AM) Standards

Various Solar Spectra

- AM0
- AM1.5G
- AM1.5 Direct + Circumsolar
- Black body at 5777 K (scaled)

Spectral Irradiance [W/m²/nm]

Wavelength [nm]

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The Earth spins from W to E, so the sun rises in E and sets in W.

- The time between two solar noon times is defined as 24 hours.
  - Note: the time of a 360° rotation of the Earth is 23 hours, 56 minutes due to Earth’s rotation around the sun.
- Observes overall symmetry
- Declination varies over course of the year.

Apparent solar motion animations

http://astro.unl.edu/naap/motion1/animations/seasons_ecliptic.html
http://astro.unl.edu/naap/motion3/animations/sunmotions.html
The relative angles of the sun and earth results in variations of energy delivery over the course of the year.
Apparent Path of the Sun

- The relative angles of the sun and earth results in variations of energy delivery over the course of the year.
Solar Radiation Reaching Earth

• The radiative energy from the sun actually reaching the Earth can be calculated:

• Areas of sun/earth:

\[ A_{\text{sun}} = \frac{\pi D_{\text{sun}}^2}{4} \quad A_{\text{earth}} = \frac{\pi D_{\text{earth}}^2}{4} \]

• View factors of sun/earth:

\[ F_{e \rightarrow s} = \frac{A_{\text{earth}}}{R_{se}^2} = 5.667 \times 10^{-9} \]
\[ F_{s \rightarrow e} = \frac{A_{\text{sun}}}{R_{se}^2} = 6.789 \times 10^{-5} \]

• Reciprocity Relation:

\[ F_{s \rightarrow e} \cdot A_{\text{sun}} = F_{e \rightarrow s} \cdot A_{\text{earth}} \]

• Calculate solar radiative flux:

\[ J_{\text{sun}} = \frac{\dot{Q}_{\text{sun}} A_{\text{sun}} F_{s \rightarrow e}}{A_{\text{earth}}} \]
\[ J_{\text{sun}} = \dot{Q}_{\text{sun}} \cdot F_{e \rightarrow s} = \varepsilon T_{\text{sun}}^4 \cdot F_{e \rightarrow s} \]

\[ J_{\text{sun}} = G_{\text{sc}} = 1366 \text{ W/m}^2 \]

Source: MIT OpenCourseWare.
Solar Radiation Reaching Earth

• The solar constant, $G_{SC} = 1366 \frac{W}{m^2}$
  – Loss Effects from day/night variation
  – Air

• Empirically: $I_{sun} = G_{SC} \times (0.7(AM^{0.678}))$
  – $I_{sun,AM0} = 956 \text{ W/m}^2$
  – $I_{sun,AM1.5} = 854 \text{ W/m}^2$
Path of the Earth

Polaris

23.45°

Earth’s equator

Ecliptic, the plane of our solar system
Local Solar Time (LST)

- Based on longitudinal position
- Sun at highest point (zenith) at solar noon
- Compare to standard (clock) time
  - Time zones limited to 24 discrete regions
  - Daylight Savings Time (DST) – artificial shift

\[ \text{LST} - \text{standard time} = 4(L_{st} - L_{loc}) + E \]

- \( L_{loc} \) = site longitude
- \( L_{st} \) = time zone standard longitude (75°W Eastern, +15° for each zone west)
Local Solar Time (LST)

- \( E = \text{Equation of Time, defined as:} \)
  - \( E = 229.2(0.000075 + 0.001868 \cdot \cos(B) - 0.032077 \cdot \sin(B) \)
    - \( 0.014615 \cdot \cos(2B) - 0.04089 \cdot \sin(2B)) \)
  - \( B = (n - 1) \frac{360}{365} \)

The difference between standard and solar time is on the order of minutes (+/- 16 minutes) This term can often be neglected unless high precision is necessary.
Hour Angle (HRA)

- Line from due north to due south, crossing through solar zenith – the **local meridian**
- **Hour Angle (HRA)** represents angular east (-) or west (+) deviation from this line.

\[ HRA = 15° \times (LST - 12) \]

- **LST** = Local solar time in fractional hours (e.g. 1:30PM = 13.5)
Declination ($\delta$)

- Day-to-day changes over the year is reflected by the **declination** ($\delta$), defined as the *angle between the noon sun and the plane of the equator*.
  - At the **equinoxes** (March 20 and September 23), tilt is not towards/away from the sun so $\delta = 0^\circ$.
  - At the solstices (June 21 and December 21), tilt maximizes at $\pm23.45^\circ$.
  - For the $n$-th day of the year:

\[
\delta = 23.45^\circ \cdot \sin \left(360^\circ \cdot \frac{284 + n}{365}\right)
\]
Factors affecting solar irradiance

• The relative positions of the sun and solar panel will dictate how much power is actually generated.

<table>
<thead>
<tr>
<th>Factor</th>
<th>Symbol</th>
<th>Dependence</th>
</tr>
</thead>
<tbody>
<tr>
<td>Latitude</td>
<td>( \Phi )</td>
<td>Geographic</td>
</tr>
<tr>
<td>Declination</td>
<td>( \delta )</td>
<td>Temporal (day)</td>
</tr>
<tr>
<td>Hour Angle</td>
<td>HRA</td>
<td>Temporal (hour)</td>
</tr>
<tr>
<td>Sun Altitude</td>
<td>( \alpha )</td>
<td>Solar</td>
</tr>
<tr>
<td>Sun Azimuth</td>
<td>( \theta )</td>
<td>Solar</td>
</tr>
<tr>
<td>Surface Tilt</td>
<td>( \beta )</td>
<td>Panel surface</td>
</tr>
<tr>
<td>Surface Azimuth</td>
<td>( \psi )</td>
<td>Panel surface</td>
</tr>
<tr>
<td>Incidence Angle</td>
<td>( \gamma )</td>
<td>Panel &amp; Solar</td>
</tr>
</tbody>
</table>
Solar Angle Effects

- **Solar Altitude, \( \alpha \)**
  - Angle from sun to ground

\[
\sin \alpha = \sin \delta \cdot \sin \Phi + \cos \delta \cdot \cos \Phi \cdot \cos(HRA)
\]

- At noon: \( \alpha = 90^\circ + \delta - \Phi \)
- Max \( \delta \) is 23°. Unless \( |\Phi| < 23^\circ \) (between Tropics of Cancer & Capricorn), sun is never overhead.
- Solar zenith angle, \( \theta_z = 90^\circ - \alpha \)
Solar Angle Effects

• Solar Azimuth, $\theta$
  - Angle between due north and a horizon point directly under the sun.
  - $N = 0^\circ$, $E = 90^\circ$, $S = 180^\circ$, $W = 270^\circ$

$$\cos \theta_r = \frac{\sin \delta \cdot \cos \Phi - \cos \delta \cdot \sin \Phi \cdot \cos(HRA)}{\cos \alpha}$$

$$\theta = \begin{cases} 
\theta_r & \text{if } HRA < 0 \\
360^\circ - \theta_r & \text{if } HRA \geq 0 
\end{cases}$$
Solar Panel Effects

• Amount of direct sunlight is dependent on both the angle of the sun and the *angle of the panel itself*.
• **Surface Elevation (tilt), $\beta$**
• Angle between surface and panel surface
Solar Panel Effects

• Amount of direct sunlight is dependent on both the angle of the sun and the \textit{angle of the panel itself}.

• \textbf{Surface Azimuth, $\psi$}

• Relative angle from north
  – Undefined $\psi$ if horizontal
  – Fixed panels in northern hemisphere are usually held due south ($\psi = 180^\circ$)
Incidence Angle ($\gamma$)

- **Incidence Angle, $\gamma$**
  - Represents *the angle between sunlight and the surface normal vector.*

- Changes with respect to time if solar panel is stationary

$$\cos \gamma = \cos \theta_z \cdot \cos \beta + \sin \theta_z \cdot \sin \beta \cdot \cos(\theta - \Psi)$$
Effect of Incidence Angle

• Amount of direct sunlight irradiance detected by the panel, $I_{\text{surface}}$, decreases as $\gamma$ increases.

$$I_{\text{surface}} = I_{\text{sun}} \cdot \cos \gamma$$

- $\gamma = 0^\circ$, $\cos(0^\circ) = 1$
- $\gamma = 15^\circ$, $\cos(15^\circ) = .966$
- $\gamma = 30^\circ$, $\cos(30^\circ) = .866$
- $\gamma = 45^\circ$, $\cos(45^\circ) = .707$
- $\gamma = 60^\circ$, $\cos(60^\circ) = .500$
- $\gamma = 75^\circ$, $\cos(75^\circ) = .259$
- $\gamma = 90^\circ$, $\cos(90^\circ) = 0$
Example Problem

- Consider a static, south-facing 2m² solar panel tilted at 10°, in Memphis TN (Lat. 35°N, Long. 90°W) on a sunny June 2nd. It is 1:30PM LST. **How much direct sunlight strikes the solar panel?**

  - $\phi = 35^\circ$, $\beta = 10^\circ$, $\psi = 180^\circ$
  - Day (n) = 31 + 28 + 31 + 30 + 31 + 2 \rightarrow **Day 154**
  - Declination ($\delta$) = $23.45^\circ \times \sin(360^\circ \times \frac{(284+154)}{365}) \rightarrow 22.3^\circ$
  - HRA = $15^\circ \times (13.5 - 12) \rightarrow 22.5^\circ$
  - Solar Altitude ($\alpha$) = $\sin^{-1}(\sin(35^\circ) \times \sin(22.3^\circ) + \cos(35^\circ) \times \cos(22.3^\circ) \times \cos(22.5^\circ)) \rightarrow 66.61^\circ$
  - Solar Azimuth ($\theta_r$) = $360^\circ - \cos^{-1}(\sin(22.3^\circ) \times \cos(35^\circ) - \cos(22.3^\circ) \times \sin(35^\circ) \times \cos(22.5^\circ)) / \cos(66.61^\circ) \rightarrow 243^\circ$
Diffuse vs. Direct Irradiance
Examples of Photovoltaic Systems

- **Single axis tracking** – in this example, aligned north-south. *Good for low latitudes.*
Examples of Photovoltaic Systems

Simple Linear

- **Single axis tracking** – in this example, aligned north-south. *Good for low latitudes.*
Examples of Photovoltaic Systems

- **Single axis azimuthal tracking** – rotation over the course of the day. *Good for high latitudes.*
Examples of Photovoltaic Systems

An azimuth tracker rotates around a vertical axis facing east in the morning, south at noon and west in the evening. It is a very effective single axis tracker especially at higher latitudes and is the standard dual-axis tracker with the addition of an elevation drive. As a single axis tracker the elevation is adjusted for the latitude during installation. Note that the view is from the south and the graphics are to be read from left to right representing the progression of a day. The sunrise and sunset positions would be for early about October and March. If this was an alt-azimuth dual-axis tracker the elevation would change from vertical early and late to the maximum elevation at noon.

- **Single axis azimuthal tracking** – rotation over the course of the day. *Good for high latitudes.*
Examples of Photovoltaic Systems

- **Dual Axis tracking** — rotation over both azimuthal and altitude. *Largely impractical configuration in practice.*
Examples of Photovoltaic Systems

Adjusted Linear Winter Adjustment

Sunrise  Mid Morning  Noon  Mid Afternoon  Sunset

Adjusted Linear Summer Adjustment

Sunrise  Mid Morning  Noon  Mid Afternoon  Sunset

• **Dual Axis tracking**  – rotation over both azimuthal and altitude. *Largely impractical configuration in practice.*